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Real Fluid Effects on an Accelerated Sphere Before Boundary-Layer Separation

S. B. MELLSEN[1]

Graduate Research Assistant, California Institute of Technology, Pasadena, Calif.

A. T. ELLIS

Associate Professor of Applied Mechanics, California Institute of Technology, Pasadena, Calif.

J. G. WAUGH[2]

Consultant, U. S. Naval Ordnance Test Station, Pasadena, Calif.

Studies were made on the apparent increase in mass on acceleration (added mass) of a sphere accelerated from rest and before boundary-layer separation, in cylinders of various diameters filled with water or oil. From a comparison of theoretical and experimentally obtained added masses, the following conclusions were drawn: In the absence of wall effects on the boundary layer, the wall shear stress over elements of the sphere can be approximated by the solution for the flat plate moving parallel to itself and the potential flow over the elements outside the boundary layer. The impulse on the elements is obtained by integration with respect to time, and the wall drag and drag impulse on the sphere by integration over the sphere surface. Good theoretical and experimental agreement obtains under the assumption that a mass of fluid, estimated from the wall drag impulse, is carried in the boundary layer and may be uniformly distributed over the sphere.

[1] **Presently, at Cambridge University, Cambridge, England.**
[2] **Also, Senior Research Fellow, California Institute of Technology, Pasadena, Calif.**

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Real Fluid Effects on an Accelerated Sphere Before Boundary-Layer Separation

S. B. MELLSEN J. G. WAUGH A. T. ELLIS

INTRODUCTION

Studies were conducted at the California Institute of Technology to determine the added mass (the apparent increase in mass on acceleration) of a 1-in-dia sphere accelerated from rest along the axes of cylinders of various diameters filled with water over short distances before boundary-layer separation and for which essentially potential flow could be expected to obtain outside the boundary layer (1).[3] It was observed, Fig.1, that the experimentally obtained added masses were consistently greater than the corresponding theoretical values computed from ideal fluid-potential theory (2). It was concluded that this experimentally observed excess in added mass must be due to real fluid effects tending to retard the acceleration of the sphere. Accordingly further studies were made to interpret these effects. These studies are described herein.

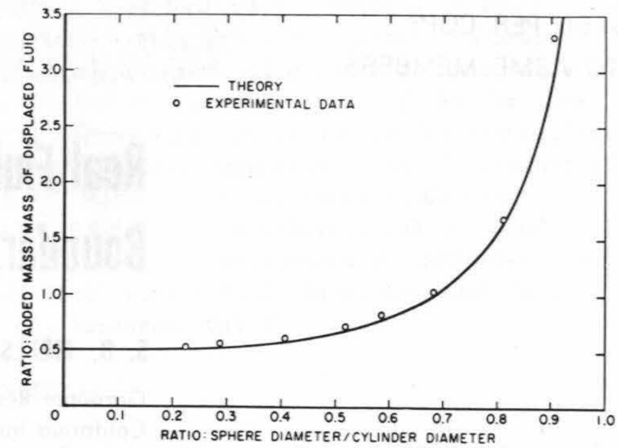

Fig.1 Added mass of a sphere accelerated from rest along axis of a circular cylinder filled with water. Experimental data were obtained before boundary-layer separation and are not corrected for real fluid effects. The theoretical data were computed from reference (2)

APPARATUS AND EXPERIMENTAL TECHNIQUES

A highly polished hollow steel sphere was used in these and the previous studies. It was 1.004 ± 0.0001 in. in diameter and weighted so that it was very slightly negatively buoyant. It was positioned below the center and on the axis of a horizontal electromagnetic coil and accelerated vertically upward by discharging a bank of heavy-duty capacitors through the coil. An Ignitron mercury switch in the circuit prevented current oscillation and limited the electromagnetic effect to a half-wave sinusoidal pulse. The duration of the pulse was about 4 millisec and the displacement of the sphere over the pulse regime about 1/8 in. or less. By accelerating the sphere in fluid and in air under otherwise similar conditions, it was possible to determine the added mass of the sphere.

A diagram of the electromagnetic acceleration apparatus is shown in Fig.2. For launchings in liquids the coil and cylinder were axially concentric and the sphere was positioned on the axis of the cylinder at a suitable operating distance below the coil by means of a suspending thread. Since the negative buoyancy of the sphere was negligible, it was not necessary to take into consideration any stored energy in the thread in the impulse to the sphere. For calibration launchings in air where this would not be the case and estimation of the energy in the thread would be uncertain, the sphere was supported by a lucite rod. Because of hydrodynamic interference, a supporting rod could not be used in the tests with liquids. An optical technique (3) was used to obtain sphere displacement-time data. In order to minimize optical distortion effects, the cylinder was placed in a lucite water tank (4) and the water level adjusted so that it was just below the accelerating coil. A more complete description of the electromagnetic accelerating technique together with the theory involved will be published later. From theory, the magnetic force, F_Z, (directed vertically upward) on the sphere, is given by

$$F_z = \frac{Kze^{-2ct}\sin^2 \omega t}{(R_c^2 + z^2)^4} \qquad (1)$$

[3] Underlined numbers in parentheses designate References at the end of the paper.

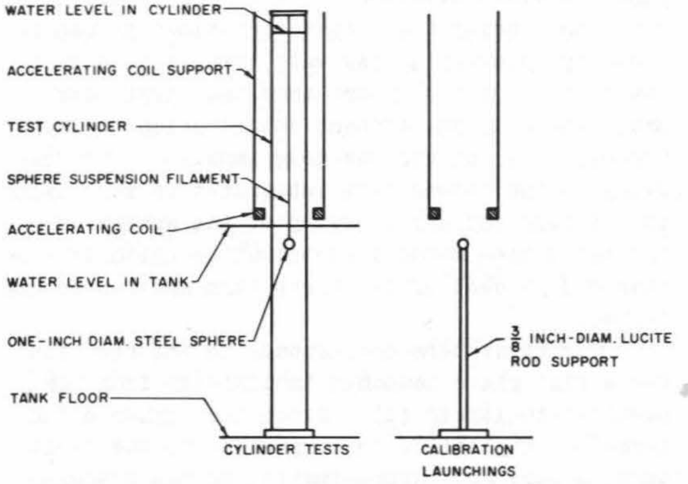

Fig. 2 Techniques used to accelerate sphere in cylinder filled with water or oil and in air

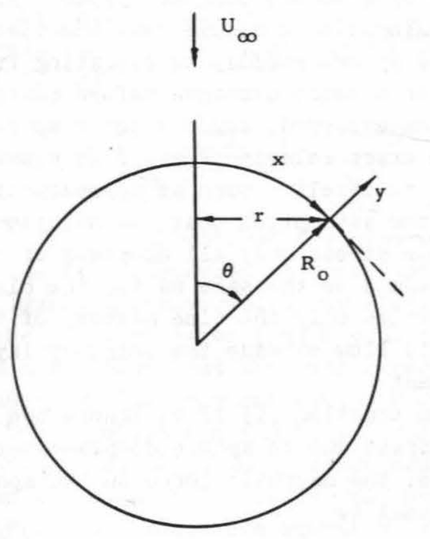

Fig. 3 Coordinate system for boundary-layer flow on a sphere

where R_c is the coil radius, z the distance from the sphere center to coil center, α the damping constant, ω the natural frequency of the circuit, and t the time, and K is a constant of proportionality.

EXPERIMENTAL AND THEORETICAL STUDIES

Experiment

In order to increase the real fluid effects and thereby facilitate their observation and interpretation, it was decided in these tests to use a fluid of considerably greater viscosity than water. Accordingly No. 7 white mineral oil of 0.8806 gm cm^{-3} density and 63.61 $\pm$ 0.10 centipoises viscosity at 21 C as measured with a Hoeppler precision viscometer was selected. The sphere was pulsed vertically upward from rest along the axis of a 4.45-in-id lucite cylinder filled with the mineral oil. The surface of the oil and bottom of the cylinder were both about 20 in. from the center of the sphere, thereby avoiding bottom and fluid surface effects. The tests were made at 21 C, the temperature at which the viscosity of the oil was measured.

Eight individual tests were made to obtain the required data. The results are shown in Table 1. To compute the added mass a similar set of

Table 1 One-Inch-Diameter Sphere Accelerated in No. 7 Oil in 4.45-In-id Cylinder

Test No.	U_∞ (in. $sec.^{-1}$)	Pulse Duration T/2 (sec.)
550	51.75	4.0324×10^{-3}
551	51.60	4.0292
552	51.92	4.0274
553	51.85	4.0260
554	51.90	4.0285
555	51.41	4.0240
556	51.89	4.0295
557	52.01	4.0312
Mean Value	$51.79 \pm 0.07^*$	4.029×10^{-3}

U_∞ - final velocity of sphere at end of force pulse

* - standard error of mean

Table 2 One-Inch-Diameter Sphere Accelerated in Air

Test No.	U_a (in. $sec.^{-1}$)	Pulse Duration T/2 (sec.)
558	81.04	4.0131×10^{-3}
559	80.52	4.0207
560	79.89	4.0161
561	79.63	4.0254
562	79.39	4.0143
563	79.78	4.0256
564	79.15	4.0087
565	79.52	4.0177
Mean Value	$79.87 \pm 0.22^*$	4.018×10^{-3}

U_a - final velocity of sphere at end of force pulse

* = standard error of mean

tests were made in air. The results are shown in Table 2.

Theory

Assuming coordinates as shown in Fig.3, the fundamental boundary-layer equations for flow past a sphere (1,5) are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial (ur)}{\partial x} + \frac{\partial (vr)}{\partial y} = 0 \qquad (2)$$

The wall shear stress, σ_{xy}, on a sphere launched impulsively from rest at velocity U_∞ at time $t = 0$, can be derived (5) from these equations. That is

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U}{2\sqrt{\nu t}}\left\{\zeta_o''(0) + t\left[\frac{dU}{dx}\zeta_{1a}''(0) + \frac{U}{r}\frac{dr}{dx}\zeta_{1b}''(0)\right] + t^2\left[\left(\frac{dU}{dx}\right)^2\zeta_{2a}''(0) + \frac{Ud^2U}{dx^2}\zeta_{2b}''(0) + \frac{U}{r}\frac{dU}{dx}\frac{dr}{dx}\zeta_{2c}''(0) + \frac{U^2}{r}\frac{d^2r}{dx^2}\zeta_{2d}''(0) + \frac{U^2}{r^2}\left(\frac{dr}{dx}\right)^2\zeta_{2e}''(0)\right] + \ldots\right\} \qquad (3)$$

where x and y are the coordinates along the surface and perpendicular to the surface, respectively.

U = potential flow velocity at surface of sphere
u = velocity of viscous fluid around sphere
v = velocity of viscous fluid perpendicular to sphere
r = perpendicular distance from axis to surface of sphere
μ = dynamic viscosity of fluid
ν = kinematic viscosity of fluid
$\zeta_{\alpha\beta}''(0)$ = numerical coefficients of series solution to boundary-layer equations evaluated by E. Voltze in his Gottingen thesis (5)

$\zeta_o''(0) = 1.128$	$\zeta_{2b}''(0) = -0.068$
$\zeta_{1a}''(0) = 1.614$	$\zeta_{2c}''(0) = -0.029$
$\zeta_{1b}''(0) = 0.169$	$\zeta_{2d}''(0) = -0.022$
$\zeta_{2a}''(0) = -0.248$	$\zeta_{2e}''(0) = 0.036$

For the tests described in this paper, the maximum (final) velocity $U = 51.71$ in. sec^{-1} and time $t = 4.028$ millisec. The sphere radius $R_o = 0.502$ in. Using these test conditions it can be shown by integrating the wall shear stress over the surface of the sphere that the first term makes the only significant contribution to the viscous drag and viscous-drag impulse. The drag owing to the second term integrates to zero owing to the fore and aft symmetry of the sphere and the third term makes a contribution which is less than 0.8 percent of the first term and can be neglected.

The first term corresponds to the solution for a flat plate launched impulsively from rest parallel to itself (1). Since the higher order terms are negligible in comparison to the first term, a very good approximation of the viscous wall drag can be found by considering only this term. Therefore the wall shear stress is very nearly directly proportional to the potential flow velocity outside the boundary layer. From this it is reasonable to assume that the viscous wall drag of a sphere rapidly accelerating from rest, taken over a short distance before boundary-layer separation occurred, could also be approximated from the exact solution for a flat plate moving parallel to itself. Such an approximation would involve the assumption that the solution for the wall shear stress over all elements of the sphere surface would be the same as for the plate and would involve only the time history of the potential fluid flow outside the boundary layer over the element.

From equation (1) if we ignore the very slight effect due to sphere displacement during the pulse, the magnetic force on the sphere is proportional to

$$e^{-2at}\sin^2\omega t \qquad (4)$$

In order to derive an expression for the velocity of the sphere we assume initially viscous forces are small in comparison to the magnetic force (the negative-buoyancy force is negligible). Later we will introduce a correction. Then the velocity, $U_\infty(t)$, of the sphere is given by

$$U_\infty(t) = K\int_0^t e^{-2at}\sin^2\omega t\,dt \qquad (5)$$

where K is a constant of proportionality. The value of K can be determined from the known sphere velocity at the end of the force pulse in the experimental tests

$$U_\infty\left(\frac{T}{2}\right) = K\int_0^{T/2} e^{-2at}\sin^2\omega t\,dt \qquad (6)$$

Then

$$K = \frac{U_\infty(\frac{T}{2})}{\int_0^{T/2} e^{-2at}\sin^2\omega t\,dt} \qquad (7)$$

and

$$U_\infty(t) = U_\infty(\frac{T}{2})\frac{\int_0^{t} e^{-2at}\sin^2\omega t\,dt}{\int_0^{T/2} e^{-2at}\sin^2\omega t\,dt} \qquad (8)$$

Then the potential fluid flow over an element of the sphere surface would be given by

$$U(\theta, t) = U(\theta)U_\infty(t) = U(\theta)U_\infty(\frac{T}{2})\frac{\int_0^{t} e^{-2at}\sin^2\omega t\,dt}{\int_0^{T/2} e^{-2at}\sin^2\omega t\,dt} \qquad (9)$$

where $U(\theta)$ is a function of the angle θ, Fig. 3.

Now the solution to the problem of the wall shear stress for the flat plate moving parallel to itself is (6)

$$\sigma_{xy} = \mu\int_0^t \frac{f'(\tau)}{\sqrt{\pi\nu(t-\tau)}}d\tau + \frac{\mu f(0)}{\sqrt{\pi\nu t}} \qquad (10)$$

where $f(t)$ is the velocity of the plate with respect to the fluid, assumed at rest, outside the boundary layer, or conversely, the fluid velocity outside the boundary layer over the plate. Now under the assumption that the wall shear stress of the sphere-surface element would be approximated by the solution for the flat plate where the potential fluid flow outside the boundary layer over the element is considered, we set $U(\theta, t) = f(t)$ and substitute equation (9) into equation (10). We obtain for the wall shear stress on the element at any time t during the force pulse

$$\sigma_{xy} = \frac{U(\theta)U_\infty(\frac{T}{2})\rho\sqrt{\frac{\nu}{\pi}}\int_0^t \frac{e^{-2a\tau}\sin^2\omega\tau\,d\tau}{\sqrt{t-\tau}}}{\int_0^{T/2} e^{-2a\tau}\sin^2\omega\tau\,d\tau} \qquad (11)$$

$$0 \le t \le T/2$$

The wall shear-stress impulse on the element over the pulse period is

$$I_{\sigma_{xy}} = \int_0^{T/2}\sigma_{xy}\,dt$$

$$= \frac{2U(\theta)U_\infty(\frac{T}{2})\rho\sqrt{\frac{\nu}{\pi}}\int_0^{T/2}\sqrt{\frac{T}{2}-\tau}\,e^{-2a\tau}\sin^2\omega\tau\,d\tau}{\int_0^{T/2} e^{-2a\tau}\sin^2\omega\tau\,d\tau} \qquad (12)$$

Then it is only necessary to determine the potential flow velocity distribution, $U(\theta)$, over the sphere and integrate equations (11) and (12) over the sphere surface to obtain the wall drag and wall-drag impulse.

The free-stream velocity, V, for flow about a sphere in a circular cylinder is given by

$$V = (\nabla \times \vec{A_\phi})_\theta \qquad (13)$$

where $\overline{A_\phi}$ is the vector potential given in reference (2). From equation (13) the potential flow velocity over the sphere was computed and $U(\theta)$ found to be

$$U(\theta) = \sin\theta\left[1 + \frac{C_o}{3} + 0.5312\left(\frac{R_o}{a}\right)^3 C_o - 0.2001\left(\frac{R_o}{a}\right)^5 C_o + 1.0005\left(\frac{R_o}{a}\right)^5 C_o\cos^2\theta + \ldots\right] \qquad (14)$$

R_o = radius of sphere
a = radius of cylinder
C_o = constant coefficient depending upon R_o/a

DISCUSSION

For the experimental tests performed in the laboratory

$$\frac{R_o}{a} = 0.226, \quad C_o = 1.5157$$

giving from equation (14)

$$U(\theta) = 1.5144\sin\theta + 0.0009\cos^2\theta\sin\theta \qquad (15)$$

The drag and drag impulse for the sphere launched along the axis of the cylinder are now found using equations (11), (12), and (15). The sphere drag at t = T/2 owing to wall shear stress is

$$D = \frac{\rho\sqrt{\frac{\nu}{\pi}}\int_0^{\pi} 2\pi R_o^2\sin^2\theta\,U(\theta)\,d\theta\,U_\infty(\frac{T}{2})\cdot\int_0^{T/2}\frac{e^{-2c\tau}\sin^2\omega\tau\,d\tau}{\sqrt{\frac{T}{2}-\tau}}}{\int_0^{T/2} e^{-2a\tau}\sin^2\omega\tau\,d\tau} \qquad (16)$$

$$= 1.001 \pm 0.002 \times 10^4 \text{ gm.in.sec.}^{-2}$$

The units are mixed for convenience here. In the laboratory the scale used for weighing was calibrated in grams and measurements of distance were made in inches.

The sphere-drag impulse owing to wall shear stress over the propulsive pulse period is from equation (12)

$$I_D = \frac{2\rho\sqrt{\frac{\nu}{\pi}} \int_0^{\pi} 2\pi R_o^2 \sin^2\theta\, U(\theta)\, d\theta\, U_\infty(\frac{T}{2}) \cdot \int_0^{T/2} \sqrt{\frac{T}{2} - \tau}\; e^{-2\alpha\tau} \sin^2 \omega\tau\, d\tau}{\int_0^{T/2} e^{-2\alpha\tau} \sin^2 \omega\tau\, d\tau} \quad (17)$$

$$= 39.31 \pm 0.07 \text{ gm. in. sec.}^{-1}$$

The integrals in equations (16) and (17) were evaluated numerically using Simpson's rule.

Further analysis (to be discussed later) showed that the magnetic impulse in oil was 750. 235 gm in. sec^{-1} and it was inferred that wall shear-stress drag significantly affected the expression for the velocity of the sphere. Using the coefficients 750.235/(750.235 - 39.31) = 1. 055 and 39.31/(750.235 - 39.31) = 0.055, a second approximation for the sphere velocity was formulated

$$U_\infty(t) = U_\infty(\frac{T}{2}) \left[1.055 \frac{\int_0^t e^{-2\alpha\tau} \sin^2 \omega\tau\, d\tau}{\int_0^{T/2} e^{-2c\tau} \sin^2 \omega\tau\, d\tau} - 0.055 \frac{\int_0^t \sqrt{t - \tau}\; e^{-2\alpha\tau} \sin^2 \omega\tau\, d\tau}{\int_0^{T/2} \sqrt{\frac{T}{2} - \tau}\; e^{-2\alpha\tau} \sin^2 \omega\tau\, d\tau} \right] \quad (18)$$

Substituting equation (18) in place of equation (8) in the previous analysis, the following values for t = T/2 were obtained:

$$D = 0.967 \pm 0.02 \times 10^4 \text{gm. in. sec.}^{-2} \quad (19)$$

$$I_D = 39.46 \pm 0.07 \text{ gm. in. sec.}^{-1} \quad (20)$$

The values show little change from those initially obtained and indicate no further approximation was needed.

For the preceding analysis to be valid it is necessary that boundary-layer separation did not occur. The maximum (final) Reynolds number is 4.64×10^2, well below the critical (3×10^5), and laminar boundary-layer flow should obtain. E. Boltze showed (1) that the distance a sphere launched impulsively from rest travels before separation starts is S = 0.392 R_o. H. Blasius (1) showed in the case of two-dimensional flow that separation occurs at longer distances from the starting point for constant acceleration than for motion started impulsively. In brief, motion started impulsively appears to be the worst case. In these tests R_o = 0.502 in. and if the motion were started impulsively S would be 0.197 in. From data measurements the displacement of the sphere during the force pulse was 0.112 in., and since its acceleration was a damped half-sine wave with respect to time, the sphere must have travelled a considerable distance after pulse termination before separation began.

For the sphere launched in the viscous fluid, equating the total impulse acting on the sphere to its momentum change gives

$$\int_0^{T/2} (F_o - D - G_o)\, dt = \int d(M' U_\infty) = \Big[(M + m) U_\infty \Big]_{t=0}^{t=T/2} \quad (21)$$

where

F_o = magnetic propulsive force in oil
D = viscous-drag force owing to wall shear stress
G_o = negative-buoyancy force in oil
M = mass of sphere
m = added mass
U_∞ = velocity of sphere through fluid

For a sphere launched in air the impulse momentum relationship is

$$\int_0^{T/2} (F_a - G_a)\, dt = MU_a \quad (22)$$

where

U_a = velocity of sphere in air at t = T/2
F_a = magnetic propulsive force in air
G_a = gravitational force on sphere.

Defining the impulses by

$$I_a = \int_0^{T/2} F_a\, dt \quad (23)$$

and using equations (21) and (22), the following relationship can be deduced:

$$\frac{m}{M} = \frac{1}{U_\infty} \left[\frac{U_a + \frac{I_{G_a}}{M}}{\left(\frac{I_{F_a}}{I_{F_o}}\right)} - \frac{I_D}{M} - \frac{I_{G_o}}{M} \right] - 1 \quad (24)$$

Now from equation (1) the magnetic propulsive force on the sphere is not only a function of time but also a function of sphere position. Since the acceleration of the sphere will be greater in air than in oil, its displacement toward the coil for corresponding times during the pulse period will be greater, and its impulse greater. This difference in impulse was taken into account by measuring the sphere displacement-time records and numerically integrating the data. The impulse ratio for the sphere in air and in oil was found to be

$$\frac{I_{F_a}}{I_{F_o}} = 1.0093 \tag{25}$$

The pulse duration, $T/2$, was about 4.02 millisec and $g = 386.1$ in. sec^{-2}. For launchings in air, the sphere left the supporting rod 0.08 millisec after pulse initiation, therefore $I_{G_a}/M = 1.52$ in. sec^{-1}. The correction to the magnetic-force impulse because of this was found to be negligible. Now $M = 9.3035$ gm and $G_o = 1.6567$ gm, from which $I_{G_o} = 0.276$ in. sec^{-1}. [The magnetic-force impulse in oil is given by $(MU_a + I_{G_a})/(I_{F_a}/I_{F_o}) = 750.235$ gm in. sec^{-1}.] Substituting these values, I_D from equation (20), the impulse ratio from equation (25) and the values of U_a and U_∞ from Tables 1 and 2 into equation (24), we obtain

$$\frac{m}{M} = \frac{1}{51.79}\left(\frac{79.87 + 1.52}{1.0093} - 4.241 - 0.276\right) - 1 = 0.4698 \pm 0.0061 \tag{26}$$

From potential theory (2) the added-mass ratio of a 1-in. sphere in ideal fluid in a 4.45-in-dia cylinder is found to be

$$\frac{m}{\rho V} = C_o - 1 = 0.5157 \tag{27}$$

where ρ is the density of the fluid, V the volume of fluid displaced by the sphere, and m the added mass.

When the sphere moves in a viscous fluid its boundary layer grows and the moving body inside the potential flow increases in size. This in turn would cause the added mass to increase. If we assume that the sphere with its boundary layer is still a sphere and its increase in size does not sensibly affect the foregoing theoretical value determined by the sphere to cylinder-diameter ratio, the added-mass ratio becomes

$$\frac{m}{\rho V + M_D} = 0.5157 \tag{28}$$

where M_D is the mass of fluid carried along in the boundary layer of the sphere due to shear force in the fluid, and $\rho V + M_D$ is the mass of fluid displaced by the sphere with its boundary layer.

The quantity M_D can be determined by considering the drag impulse from equation (20). Equating the drag impulse I_D to its corresponding momentum change, we have

$$I_D = \int_0^{T/2} D\,dt = \int_0^{T/2} d(M_D U_\infty) \tag{29}$$

or

$$I_D = M_D U_\infty \Big|_0^{T/2} = M_D\left(\frac{T}{2}\right) U_\infty\left(\frac{T}{2}\right) \tag{30}$$

the momentum of fluid carried along with the sphere. Since the drag impulse I_D is given by equation (20) and the velocity of the sphere U_∞ is known at the end of the force pulse, the mass M_D can be found. Then

$$M_D = \frac{I_D}{U_\infty\left(\frac{T}{2}\right)} = \frac{39.46 \pm 0.07 \text{ gm. in. sec.}^{-1}}{51.79 \pm 0.07 \text{ in. sec.}^{-1}} = 0.7619 \pm 0.0024 \text{ gm.} \tag{31}$$

The volume of the sphere $V = 8.6836$ cu cm so that

$$\rho V + M_D = 7.6468 \pm 0.0017 \text{ gm.} + 0.7619 \pm 0.0024 \text{ gm.} = 8.4087 \pm 0.0041 \text{ gm.} \tag{32}$$

Now

$$\frac{M}{\rho V + M_D} = \frac{9.3035 \pm 0.0001 \text{ gm.}}{8.4087 \pm 0.0041 \text{ gm.}} = 1.1064 \pm 0.0005 \tag{33}$$

Then from equations (26) and (33)

$$\frac{m}{\rho V + M_D} = \left(\frac{m}{M}\right)\left(\frac{M}{\rho V + M_D}\right) = 0.5198 \pm 1.4\% \tag{34}$$

Here the possible error in the impulse correction has not been accounted for. Since the total correction is less than 3 percent of the added-mass coefficient allowing a generous error of 15 percent in the correction would still keep the total experimental error under ± 2 percent.

Comparison of the experimentally evaluated added-mass ratio and its corresponding theoretical value gives

$$\frac{m_{\text{experimental}}}{m_{\text{theoretical}}} = \frac{0.5198}{0.5157} = 1.0080 \tag{35}$$

a very close agreement, well within the estimated total experimental error. Failure to take into consideration either I_D and/or M_D would cause a large divergence in the experimental added-mass ratio.

We will consider now the effect of increased sphere size and hence, increased sphere-cylinder radius ratio. For a mass of fluid (oil) $M_D = 0.7619$ gm, carried along in the boundary layer, the corresponding volume of fluid would be 0.8652 cu cm = 0.0528 cu in. This volume, uniformly distributed, would increase the sphere diameter to 1.037 in., giving a ratio $R_o/a = 0.2328$ for which the theoretical added-mass ratio is 0.5173. This would give an experimental to theoretical added-mass ratio of 1.0048, an even better agreement. Finally, the determination of the experimental to theoretical added-mass ratio at the end of the pulse regime is quite arbitrary and very good agreement should obtain for all distances over the regime, and, presumably, all distances before boundary-layer separation occurs.

The method described in the foregoing was then applied to the tests with water where the dynamic viscosity was about 1/64 that of the oil. The mean correction for real fluid effects in the tests in the seven largest cylinders was 2.83 percent of the theoretical value, giving for these cylinders a mean ratio of experimental to theoretical added mass of 0.9902, a very good agreement. For the smallest cylinder the interaction of the boundary layer around the sphere with the cylinder wall is very complicated and no quantitative evaluation of its effect on the added mass was made.

Study of the sphere motion in oil after termination of the pulse regime indicated a good agreement, until boundary-layer separation, between the viscous drag evaluated from boundary-layer theory and that evaluated directly from experiment.

CONCLUSIONS

The following conclusions are drawn in regard to real fluid effects on a sphere accelerated from rest in the absence of wall effects on the boundary layer and prior to boundary-layer separation:

1 The wall shear stress over elements of the sphere surface can be approximated by the solution for the sliding flat plate and the potential flow over the elements outside the boundary layer. The impulse on the elements is obtained by integration with respect to time, and the drag and drag impulse on the sphere by integration of these quantities over the sphere surface.

2 A mass of fluid, estimated from the wall-drag impulse, may be carried along in the boundary layer and can be assumed to be uniformly distributed over the sphere surface.

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